

Tidal satellite perturbations and the Lense-Thirring effect

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Abstract

The tiny general relativistic Lense-Thirring effect can be measured by means of a suitable combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. This observable is affected, among other factors, by the Earth's solid and ocean tides. They induce long-period orbital perturbations that, over observational periods of few years, may alias the detection of the gravitomagnetic secular trend of interest. In this paper we calculate explicitly the most relevant tidal perturbations acting upon LAGEOSs and assess their influence on the detection of the Lense-Thirring effect. The present day level of knowledge of the solid and ocean tides allow us to conclude that their influence on it ranges from almost 4% over 4 years to less than 2% over 7 years.

1. Introduction

In the past few years we have seen increasing efforts, both from a theoretical and an experimental point of view, devoted to the measurement, by means of artificial satellites techniques, of certain very small general relativistic effects in the weak gravitational field of the Earth. We refer to the small forces, unknown in classical Newtonian mechanics, induced by the rotation of the Earth itself on near orbiting bodies; due to their formal analogy with the magnetic forces generated by electric charge currents they are called gravitomagnetic effects. Among them we recall the dragging of the orbital plane of a point mass and the shift of the pericenter's orbit in its plane, called Lense-Thirring effect (Lense and Thirring 1918; Ciufolini and Wheeler 1995), which is presently measured by analyzing the orbits of LAGEOS and LAGEOS II SLR satellites (Ciufolini 2000).

In the LAGEOS experiment the observable is a suitable combination of the orbital residuals of the nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II (Ciufolini 1996):

$$\delta\dot{\Omega}^I + c_1\delta\dot{\Omega}^{II} + c_2\delta\dot{\omega}^{II} \simeq 60.2\mu_{LT}, \quad (1)$$

in which $c_1 = 0.295$, $c_2 = -0.35$ and μ_{LT} is the parameter which must be measured. It is 1 in general relativity and 0 in Newtonian mechanics. General relativity predicts for the combined residuals a linear trend with a slope of 60.2 mas/y. Indeed:

$$\dot{\Omega}_{LT}^{LAGEOS} \simeq 31 \text{ mas/y}, \quad (2)$$

$$\dot{\Omega}_{LT}^{LAGEOSII} \simeq 31.5 \text{ mas/y}, \quad (3)$$

$$\dot{\omega}_{LT}^{LAGEOSII} \simeq 57 \text{ mas/y}. \quad (4)$$

In order to assess the feasibility of this space-based experiment the error budget is of the utmost importance, in particular with respect to the systematic uncertainties induced by other physical forces acting upon the combined residuals of eq.(1). Among them the orbital perturbations induced by the Earth's solid and ocean tides (Dow 1988; Christodoulidis et al. 1988; Casotto 1989) are very relevant. Ciufolini claims that eq.(1) is not affected by the $l = 2, 4$ $m = 0$ part, both statical and dynamical, of the Earth's gravitational field. If confirmed, this feature is very important since the systematic errors of the terrestrial gravitational field which are most relevant for the detection of the Lense-Thirring effect lie just in the first two even zonal harmonics (Ciufolini 1996).

In order to model as more accurately as possible the tidal perturbations on the LAGEOSs and to test quantitatively the independence of the proposed combined residuals of the $l = 2, 4$ $m = 0$ harmonics a complete analysis of the Earth tidal perturbations (Pavlis and Iorio 2000) on the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II is carried out. The obtained results are used in assessing the level at which the poor knowledge of the Earth's solid and, in particular, ocean tides affects the measurement of μ_{LT} by means of eq.(1).

2. Earth's solid and ocean tidal perturbations on LAGEOSs' orbits

The tidal perturbations are worked out in the framework of the linear Lagrange perturbation theory (Kaula 1966); since the Lense-Thirring effect is a linear trend, only the long-period perturbations averaged over an orbital revolution are considered.

The investigation of the tidal perturbations involves only the terms of degree $l = 2$ for the solid tides and also the terms of degree $l = 3, 4$ for the ocean tides. Concerning the latter ones, only the prograde waves are considered.

The perturbative amplitudes are inversely proportional to the perturbations' frequencies. Concerning the examined even degree perturbations, the periods depend only on the relative motion, as viewed in a geocentric inertial frame, between the constituent's tidal bulge and the orbital plane. Concerning the odd degree ocean tidal perturbations, also the perigee's motion must be considered. So, in general, the spectrum of the orbital tidal perturbations is richer than that of the tides as viewed on the Earth's surface: often it happens that a constituent which on the Earth gives rise to negligible tides induces orbital perturbations with relevant amplitudes and different periods.

In general, as far as the solid and ocean tides' constituents whose perturbations are examined here, it must be noted that, for $l = 2$, the ocean perturbative amplitudes amount to at most 10% of the corresponding solid perturbative amplitudes.

The calculations are performed by using for the Love numbers k_2 the complex, frequency-dependent values released by (McCarthy 1996), for the equilibrium tidal heights H_m^l the values released by (Roosbeek 1996) and for the ocean tidal heights C_{lmf}^+ the coefficients of the recent EGM96 model (Lemoine et al 1998).

2.1. The solid tides

Concerning the solid tides, the most effective constituents turn out to be the semisecular 18.6-year, the K_1 and the S_2 which induce on the examined LAGEOSs' elements perturbations of the order of $10^2 - 10^3$ mas with periods ranging from 111.24 days for the S_2 on LAGEOS II to 6798.38 days for the 18.6-year tide.

The latter, with its amplitudes of -1079.38 mas, 1982.16 mas and -1375.58 mas for the nodes of the LAGEOSs and the perigee of LAGEOS II respectively, could be particularly insidious for the detection of the Lense-Thirring effect. Indeed, since the observational periods adopted until now (Ciufolini 2000) range only from 3.1 to 4 years, it could resemble a superimposed linear trend which may alias the recovery of μ_{LT} . However, its effect should vanish since it is a $l = 2$, $m = 0$ tide, and eq.(1) should be not sensitive to such tides. This feature will be quantitatively assessed later.

Also the K_1 cannot be neglected: it induces on the LAGEOS' s node a perturbation with a period of 1043.67 days and an amplitude of 1744.38 mas, while the node and the perigee of LAGEOS II are shifted by an amount of -398 mas and 1982.14 mas, respectively, with a period of -569.21 days.

2.2. The ocean tides

Regarding the ocean tides, whose knowledge is less accurate than that of the solid tides, the $l = 3$ part of the tidal spectrum turns out to be very interesting for the perigee of LAGEOS II.

Indeed, for this element the K_1 $l = 3$ $p = 1$, 2 $q = -1$, 1 terms induce perturbations whose amplitudes and periods are of the same order of magnitude of those generated by the solid tides. For example, the $l = 3$ $p = 1$ $q = -1$ harmonic has a period of 1851.9 days (5.09 years) and an amplitude of 1136 mas, while the $l = 3$ $p = 2$ $q = 1$ harmonic is less effective: its amplitude amounts to 346.6 mas and its period to -336.28 days.

It should be noted that, contrary to the first two even degree zonal perturbations which do not affect eq.(1), the diurnal odd degree tidal perturbations are not canceled out by the combined residuals of Ciufolini. So, over an observational period of few years the K_1 $l = 3$ $p = 1$ $q = -1$ harmonic may alias the gravitomagnetic trend of interest to us.

The even degree ocean tidal perturbations are not particularly relevant: they amount to some tens of mas or less, with the exception of K_1 which perturbs the node of LAGEOS and the perigee of LAGEOS II at a level of 10^2 mas.

3. The effect of the orbital tidal perturbations on the measurement of the Lense-Thirring effect

For a given observational period T_{obs} , the tidal perturbations can be divided between two main categories according to their periods P : those with $P < T_{obs}$ and those with $P > T_{obs}$. The former ones, even if their mismodeled amplitude is great so that they heavily affect the orbital residuals, are not particularly insidious because their effect averages out

they may alias the determination of μ_{LT} acting as superimposed bias. This is particularly true for those diurnal and semidiurnal tides which should affect the combined residuals, like the K_1 $l = 3$ $m = 1$ $p = 1$ $q = -1$.

A preliminary analysis has been performed by evaluating the left-hand side of eq.(1) by means of the nominal tidal perturbative amplitudes worked out in the previous sections for $T_{obs} = 1$ year. The calculations have been repeated also by means of the mismodeled amplitudes. They show that, not only the $l = 2, 4$ $m = 0$ tides tend to cancel out, but also that this feature extends to the $l = 3$ $m = 0$ ocean tides.

A more refined procedure will be described below for the diurnal and semidiurnal tides.

3.1. Case a: $P < T_{obs}$

The effect of this class of perturbations has been assessed as follows.

The orbital residuals curve has been simulated with MATLAB by including the Lense-Thirring trend as predicted by general relativity, the main mismodeled tidal perturbations:

$$\delta A_f \sin\left(\frac{2\pi}{P_f}t + \phi_f\right), \quad (5)$$

where f denotes the chosen harmonic, and δA a noise. About the mismodeling δA we have assumed that the main source of uncertainties are the Love number k_2 (McCarthy 1996) for the solid tides, the load Love numbers (Farrell 1972; Pagiatakis 1990) and the ocean tidal heights C_{lmf}^+ (Lemoine et al. 1998) for the ocean tides. It has been decided to build into the MATLAB routine the capability to vary the time series length T_{obs} , the time span Δt , the amplitude of the noise, and the initial phases ϕ of the harmonics.

The so obtained simulated curves, for different choices of T_{obs} , have been subsequently fitted with a least-square routine in order to recover, among the other things, the parameter μ_{LT} . This procedure has been repeated with and without the whole set of mismodeled tidal signals so to obtain an estimate $\Delta\mu_{tides} = \mu_{LT}(all\ tides) - \mu_{LT}(no\ tides)$ of their influence on the measurement of μ_{LT} . This analysis shows that $2\% < \Delta\mu_{tides} < 4\%$ for T_{obs} ranging from 4 years to 7 years.

The decision of including the whole set of tides has been motivated by the fact that their least-square fitted parameters show that some of them are strongly correlated with the Lense-Thirring trend, in particular for 4 years. For longer time spans, as it could be expected, the gravitomagnetic trend emerges clearly against the background of the tidal noise and also the single harmonics, describing a large number of complete cycles, tend to mutually decorrelate.

3.2. Case b: $P > T_{obs}$

The effect of this class of perturbations has been assessed by means of different approaches.

First, in a very conservative way, the averaged value of the mismodeled tidal signal under consideration over different T_{obs} has been considered. These calculations have been performed for

tide a mismodeling level of 1.5% due to the uncertainty at its frequency in the anelastic behavior of the Earth's mantle accounted for by the Love number k_2 . The ocean constituent has been considered unknown at a 6% level due to the uncertainties in the load Love number of degree $l = 3$ and in the tidal height coefficient C_{lmf}^+ as released by EGM96. Over different T_{obs} the latter affects the determination of the Lense-Thirring effect at a 2.3% level at most, while the zonal tide, as predicted by Ciufolini (1996), almost cancels out giving rise to negligible contributions to the combined residuals.

These results have been confirmed fairly well by adopting another method analogous to that employed in (Vespe 1999) in assessing the influence of the direct solar radiation pressure's eclipses on the determination of the Lense-Thirring effect. We take the signal of interest over a given T_{obs} , fit it with a straight line and compare the so obtained slope, in unit of 60.2 mas/y, to μ_{LT} .

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References

- Casotto, S. (1989): Ocean tide models for TOPEX precise orbit determination, Ph. D. thesis, The University of Texas, Austin.
- Ciufolini, I. and J. A. Wheeler (1995): Gravitation and Inertia, Princeton University Press, New York, 498p.
- Ciufolini, I. (1996): On a new method to measure the gravitomagnetic field using two orbiting satellites, *Il Nuovo Cimento*, **A12**, 1709-1720.
- Ciufolini, I. (2000): The 1995-99 measurements of the Lense-Thirring effect using laser-ranged satellites, *Class. Quantum Grav.*, **17**, 1-12.
- Christodoulidis, D., D. E. Smith, R. G. Williamson and S. M. Klosko (1988): observed tidal braking in the Earth/Moon/Sun system, *J. Geophys. Res.*, **B6**, 6216-6236.
- Dow, J. M. (1988): Ocean Tides and Tectonic Plate Motions from Lageos, Ph. D. thesis, München.
- Farrell, W. E. (1972): Deformation of the Earth by surface load, *Rev. Geophys. Space Phys.*, **10**, 761-797.
- Kaula, W. M. (1966): Theory of Satellite Geodesy, Blaisdell Publishing Company, Waltham, 124p.
- Lemoine, F. G. et al. (1998): The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861.
- Lense, J., and H. Thirring (1918): Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, *Phys. Z.*, **19**, 156-163.
- McCarthy, D. D. (1996): IERS conventions, IERS Technical Note 21, U. S. Naval Observatory, 95p.
- Pagiatakis, S. D. (1990): The response of a realistic earth to ocean tide loading, *Geophys. J. Int.*, **103**, 541-560.
- Pavlis, E. C. and L. Iorio (2001) The impact of tidal errors on the determination of the Lense-Thirring effect from satellite laser ranging, submitted to *Int. J. of Mod. Phys. D*; Iorio, L. (1999): Earth tides and Lense-Thirring effect submitted to *Class. Mod. Phys.* /20000001

- Roosbeek, F. (1996): RATGP95: a harmonic development of the tide-generating potential using an analytical method, *Geophys. J. Int.*, **126**, 197-204.
- Vespe, F. (1999): The perturbations of Earth penumbra on LAGEOS II perigee and the measurement of Lense-Thirring gravitomagnetic effect, *Adv. Space Res.*, **23**, 4, 699-703.